Shore

Examination Number:

Set:

Year 12
Trial HSC Examination
August 2014

Mathematics

General Instructions

- Reading time − 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11−16, show relevant mathematical reasoning and/or calculations
- Start each of Questions 11−16 in a new writing booklet
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

Total marks − 100

Section I Pages 2–5

10 marks

- Attempt questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6–14

90 marks

- Attempt questions 11–16
- Allow about 2 hours and 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1−10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1−10.

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1 What is the value of $\dfrac{e^2+e^{-2}}{\sqrt{\ln 2}+1}$ correct to 3 significant figures?

(A) 4.11

(B) 5.78

(C) 7.46

(D) 7.49

2 If $\dfrac{2\sqrt{3}}{\sqrt{3}+3} = a\sqrt{3}+b$, what are the values of a and b?

(A) $a = 1, b = 1$

(B) $a = 1, b = -1$

(C) $a = -1, b = 1$

(D) $a = -1, b = -1$

3 What are the solutions of $x^2 - 4x - 2 = 0$?

(A) $x = -2 \pm \sqrt{2}$

(B) $x = 2 \pm \sqrt{2}$

(C) $x = -2 \pm \sqrt{6}$

(D) $x = 2 \pm \sqrt{6}$

4 A line has a gradient of $-\sqrt{3}$.

What is the inclination of the line to the positive x axis?

(A) 30°

(B) 60°

(C) 120°

(D) 150°

5 What is a primitive of $3 - \sin x$?

(A) $-\cos x$

(B) $\cos x$

(C) $3x - \cos x$

(D) $3x + \cos x$

6 A parabola has focus $(-4, 0)$ and directrix $x = 2$.

What is the equation of the parabola?

(A) $y^2 = -24(x+4)$

(B) $y^2 = -12(x+1)$

(C) $y^2 = 24(x+4)$

(D) $y^2 = 12(x+1)$

7 What are the solutions of $x^2 < 9$?

(A) $x < -3$ or $x < 3$

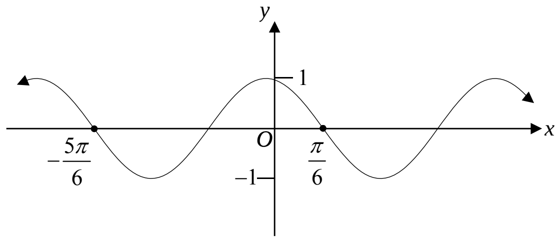
(B) $x < -3$ or $x > 3$

(C) $x > -3$ and $x < 3$

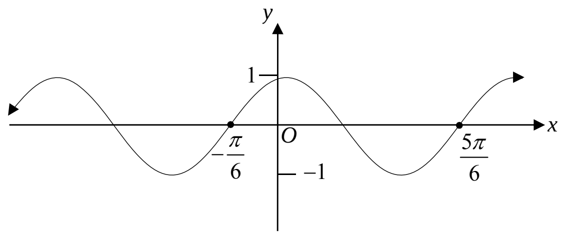
(D) $x < -3$ and $x < 3$

8 Which diagram shows the graph of $y = \cos\left(2x - \frac{\pi}{6}\right)$?

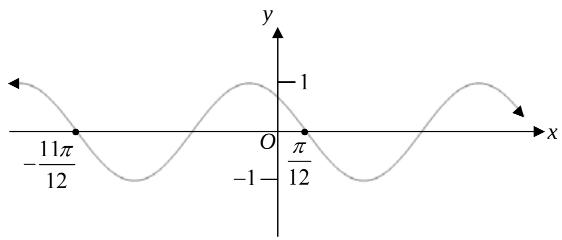
(A)



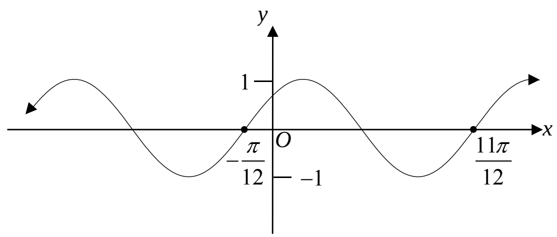
(B)



(C)



(D)



9 A particle is moving along the x-axis. The displacement of the particle after t seconds is given by $x = t^2 - 3t$ metres.

Which statement describes the motion after 1 second?

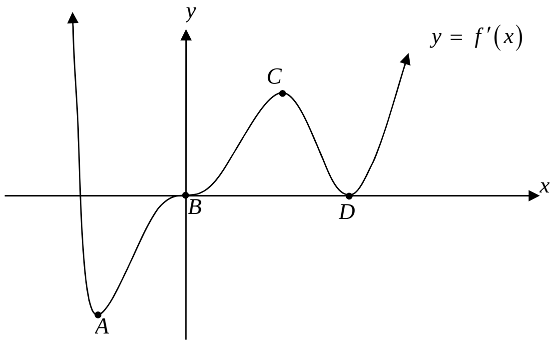
(A) The particle is moving to the left with decreasing speed.

(B) The particle is moving to the right with decreasing speed.

(C) The particle is moving to the left with increasing speed.

(D) The particle is moving to the right with increasing speed.

10 The diagram shows a sketch of the gradient function $y = f'(x)$ passing through the points A, B, C and D.



Which point represents the horizontal point of inflexion of the curve $y = f(x)$?

(A) Point A

(B) Point B

(C) Point C

(D) Point D

Section II

90 marks
Attempt Questions 11−16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11−16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $|2x + 3| < 9$. **2**

(b) Factorise $4 + 7x - 2x^2$. **2**

(c) Sketch the graph of $y = |x - 2|$. **2**

(d) Find the perpendicular distance from the point $(-3, 4)$ to the line $y = 5x$. **2**

(e) Find $f''(-2)$ if $f(x) = \log_e \sqrt{x}$. **2**

(f) Find $\int \frac{1}{x^2} + \sqrt{x}\, dx$. **2**

(g) The quadratic equation $4x^2 - 3x - 2 = 0$ has roots α and β.

(i) Find $\alpha + \beta$. **1**

(ii) Find $\alpha^3\beta^2 + \alpha^2\beta^3$. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x.

(i) xe^{x^2} **2**

(ii) $\dfrac{\sin x}{2x}$ **2**

(b) Find $\displaystyle\int \sec^2\left(\frac{x}{2}+\pi\right) dx$. **2**

(c) Evaluate $\displaystyle\int_0^2 \frac{x^3}{2+2x^4}\, dx$. Leave your answer in simplified exact form. **3**

(d) A miner is mining for a precious metal in the deserts of Western Australia. The amounts of precious metal mined in each of the first three months of operation were 4000 grams, 3920 grams, 3840 grams respectively and this pattern continues throughout the operation. The mine runs out of the precious metal after 50 months.

(i) How many grams were mined in the 12^{th} month? **1**

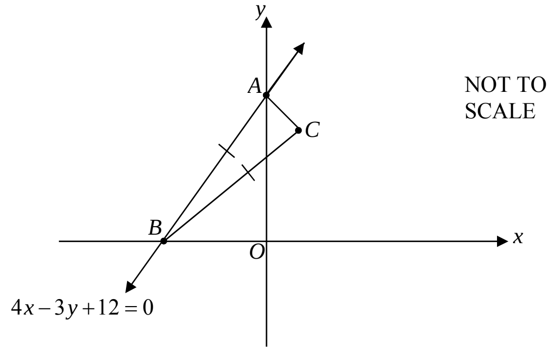
(ii) How many grams were mined over the first year? **2**

(iii) 25% of the precious metal mined each month is placed in storage for future investment. The miner sells the remaining 75% of precious metal mined each month to an overseas company. **3**

How many months does he need to mine to sell a total of 73.2 kg to the company?

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram shows triangle ABC with $AB = BC$.
The line $4x-3y+12=0$ meets the x and y axes at B and A respectively.



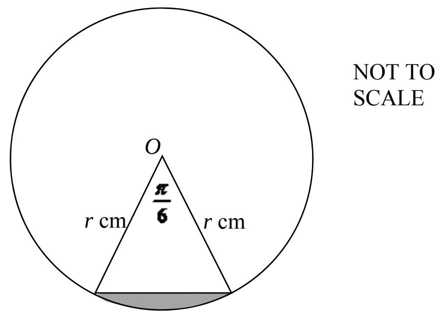
(i) Calculate the distance AB. **2**

(ii) Point C has coordinates $\left(1\frac{1}{2},\ b\right)$. Show that $b=\dfrac{\sqrt{19}}{2}$. **2**

(iii) Find the coordinates of point D if $ABCD$ forms a rhombus. **1**

(b) A circle has a minor segment with area $(3\pi-9)$ square centimetres. **3**

The segment is cut off by a chord subtending an angle of $\dfrac{\pi}{6}$ radians at the centre. Find the radius, r, of the circle.



Question 13 continues on page 9

Question 13 (continued)

(c) (i) On the same set of axes, sketch the curves $y = \sin 2x$ and $y = \cos x$ for $0 \leq x \leq \frac{\pi}{2}$. **2**

(ii) Verify that the curves intersect at $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$. **2**

(iii) Hence find the area between the two curves from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{2}$. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) In a recent poll taken on whether Australia should become a republic, the following results were obtained.

- In favour of a republic = 35%
- Against a republic = 55%
- Undecided = 10%

Two people were chosen at random from those who were surveyed.

(i) Find the probability that both were in favour of a republic. **1**

(ii) Find the probability that one would be in favour of a republic and one would be against a republic. **2**

(iii) Find the probability that at least one would be in favour of a republic. **2**

(b) The blood-alcohol content (B) after a person has been drinking is given by $B = B_0 e^{-kt}$, where B_0 represents the blood-alcohol content level at the time a person stops drinking, t is measured in hours and B in mg/ml.

Sam stops drinking at 11:00 pm on Saturday night ($t = 0$) and his blood-alcohol level was measured at 0.24 mg/ml. It took 28 hours for Sam’s blood-alcohol level to be measured at 0.001 mg/ml.

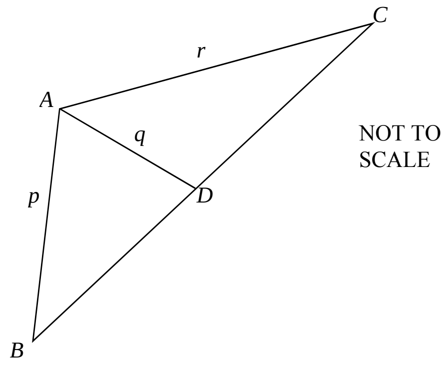
(i) Find the value of k correct to 4 decimal places. **2**

(ii) The allowable blood-alcohol level limit for Sam to drive a car is 0.05 mg/ml. What is the earliest time on Sunday that Sam will be able to legally drive? Write your answer correct to the nearest hour. **2**

(iii) What is the rate of decrease of the blood-alcohol level content in Sam’s blood at 8:00 am on Sunday? **1**

Question 14 continues on page 11

(c) In the diagram $\angle ABD + \angle BCA = \frac{\pi}{3}$, AD bisects $\angle BAC$, $AB = p$, $AD = q$, and $AC = r$.



(i) Show that $\angle BAD = \angle DAC = \frac{\pi}{3}$. **2**

(ii) By finding the areas of triangles, prove that $\frac{1}{p} + \frac{1}{r} = \frac{1}{q}$. **3**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $3\sin x \tan^2 x - \sin x = 0$ for $0 \le x \le 2\pi$. **3**

(b) The function $y = x(x-3)^2$ is defined in the domain $0 \le x \le 4$.

(i) Find the x intercepts. **1**

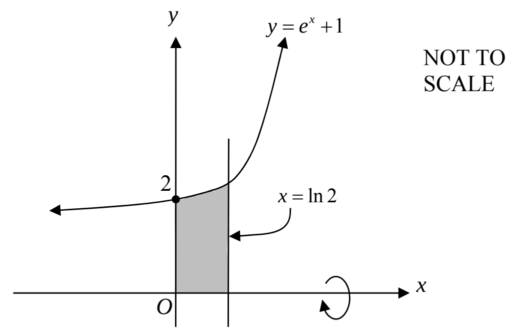
(ii) Find the coordinates of any turning points and determine their nature. **3**

(iii) Sketch the curve $y = x(x-3)^2$ for $0 \le x \le 4$, showing all essential features. **2**

(c) The limiting sum of the series $\frac{1}{p} - \frac{1}{p^2} + \frac{1}{p^3} - \ldots\ldots$ is equal to $-4p$, $(p \ne 0)$. **3**

Find the value of p.

(d)



The area bounded by the curve $y = e^x + 1$, the x axis, and the lines $x = 0$ and $x = \ln 2$ is rotated about the x axis. Find the exact volume of the solid formed. **3**

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) A particle moves in a straight line with acceleration after t seconds given by $a = 4\sin 2t$ m/s^2. Initially the particle is 1 metre to the left of the origin and travelling with a velocity of 2 m/s.

(i) Show that the velocity of the particle is given by $v = 4 - 2\cos 2t$. **2**

(ii) Show that the particle never comes to rest. **1**

(iii) Find the distance travelled by the particle in the first 4 seconds. Write your answer correct to the nearest metre. **2**

(b) Nick's grandparents have set up a fund with a single investment of \$400 000 to provide financial support for him. He is granted an annual payment of \$25 000 from this fund at the end of each year. The fund accrues interest at a rate of 5% per annum compounded annually.

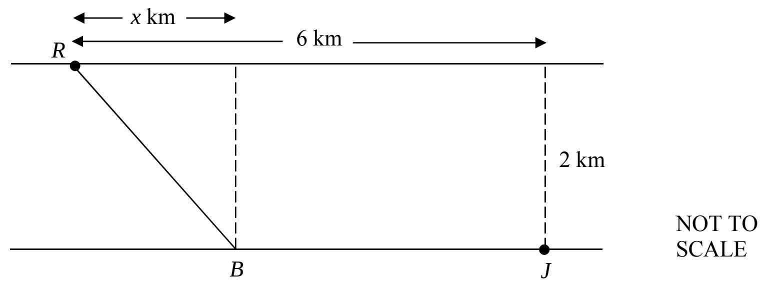
(i) Calculate the balance in the fund at the beginning of the second year. **1**

(ii) Let $\$A_n$ be the balance of the fund at the end of n years (after Nick receives his payment). Show that $A_n = 500\ 000 - 100\ 000(1.05)^n$. **2**

(iii) If this fund began at the beginning of 2000, in what year will the fund run out of money? **1**

Question 16 continues on page 14

(c)



Rick and Julie live in 2 parallel streets which are 2 kilometres apart and run east-west as shown in the diagram.

When Julie calls Rick to let him know her parents are out, he needs to get there as fast as possible. Rick has hidden a bike at point B in Julie's street.

To get to Julie's house, Rick runs from his house, R, through a park to his bike, B, at a speed of 8 km/h. He then rides to Julie's house, J at 16 km/h.

Let x kilometres represent the distance the bike is east of Rick's house.

(i) Show that the time (T hours) taken for Rick to get to Julie's house is given by **2**

$$T = \frac{\sqrt{x^2+4}}{8} + \frac{6-x}{16}.$$

(ii) Find the distance of the bike from Julie's house in order to minimize the time taken for Rick to get to Julie. **4**

End of paper

2U TRIAL HSC SOLUTIONS 2014

① $\dfrac{e^2+e^{-2}}{\sqrt{\ln 2}+1} = \dfrac{7.524\ldots}{1.8325\ldots}$

$= 4.1059\ldots$

$\doteqdot \underline{4.11}$ (A)

② $\dfrac{2\sqrt{3}}{\sqrt{3}+3} \times \dfrac{\sqrt{3}-3}{\sqrt{3}-3} = \dfrac{6-6\sqrt{3}}{3-9}$

$= \dfrac{6-6\sqrt{3}}{-6}$

$= \sqrt{3}-1$

$\therefore \underline{a=1},\ \underline{b=-1}$ (B)

③ $x^2-4x-2=0$

$x = \dfrac{4 \pm \sqrt{16-4\times1\times-2}}{2}$

$= \dfrac{4\pm\sqrt{24}}{2}$

$= \dfrac{4\pm 2\sqrt{6}}{2}$

$= \underline{2\pm\sqrt{6}}$ (D)

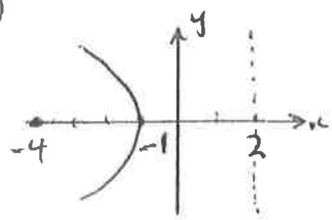
④ $\tan\alpha = -\sqrt{3}$

basic $\alpha = 60^\circ$

$\therefore \underline{\alpha = 120^\circ}$ (C)

⑤ $\int 3-\sin x \, dx$

$= \underline{3x+\cos x}$ (D)

⑥ 

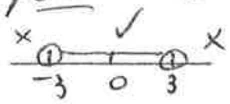
$(y-0)^2 = -4(3)(x+1)$

$\underline{y^2 = -12(x+1)}$

$\underline{a=3}$ (B)

⑦ $x^2 < 9$

Critical points $x = \pm 3$



$\therefore -3 < x < 3$

i.e. $\underline{x > -3 \text{ and } x < 3}$ (C)

⑧ $\cos\left(2x-\frac{\pi}{6}\right) = 0$ for $\underline{x \text{ intercepts}}$

$2x - \frac{\pi}{6} = \ldots -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

$2x = \ldots -\frac{7\pi}{3}, -\frac{4\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{3} \ldots$

$x = \ldots -\frac{7\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3} \ldots$

$\uparrow \qquad \uparrow$

(B)

⑨ $x = t^2-3t$

$v = 2t-3$

$\underline{a=2}$

at $\underline{t=1}$, $\underline{v=-1}$ m/s (A)

$\therefore$ moving to left with decreasing speed ✓

⑩ For H.P.O.I.

$\underline{f'(x)=0}$, $\underline{f''(x)=0}$ (D)

Question 11

(a) $|2x+3| < 9$

$-9 < 2x+3 < 9$

$-12 < 2x < 6$

$-6 < x < 3$

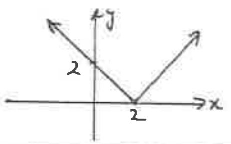
(b) $4+7x-2x^2$ P -8, S 7, F $8, -1$

$= 4+8x-x-2x^2$

$= 4(1+2x)-x(1+2x)$

$= (4-x)(1+2x)$ [or $-(x-4)(2x+1)$]

(c) $y = |x-2|$



(d) $d = \left| \frac{5(-3)-1(4)+0}{\sqrt{5^2+(-1)^2}} \right|$ $y = 5x$, $0 = 5x-y+0$

$= \left| \frac{-19}{\sqrt{26}} \right|$

$= \frac{19}{\sqrt{26}}$ [or $\frac{19\sqrt{26}}{26}$]

(e) $f(x) = \ln\sqrt{x}$

$= \frac{1}{2}\ln x$

$f'(x) = \frac{1}{2} \times \frac{1}{x}$

$= \frac{1}{2}x^{-1}$

$f''(x) = -\frac{1}{2}x^{-2}$

$f''(-1) = -\frac{1}{2}(-2)^{-2}$

$= -\frac{1}{8}$

Question 11 Continued

(f) $\int \frac{1}{x^2} + \sqrt{x}\, dx$

$= \int x^{-2} + x^{\frac{1}{2}}\, dx$

$= \frac{x^{-1}}{-1} + \frac{2}{3}x^{\frac{3}{2}} + C$

$= -\frac{1}{x} + \frac{2}{3\sqrt{x^3}} + C$

(g) (i) $\alpha+\beta = -\frac{b}{a}$

$= \frac{3}{4}$

(ii) $\alpha^3\beta^2 + \alpha^2\beta^3 = \alpha^2\beta^2(\alpha+\beta)$

$= (\alpha\beta)^2(\alpha+\beta)$

$= \left(-\frac{1}{2}\right)^2\left(\frac{3}{4}\right)$

$= \frac{1}{4} \times \frac{3}{4}$

$= \frac{3}{16}$

Question 12

(a) (i) $y = xe^{x^2}$

$y' = e^{x^2} \times 1 + x \times 2xe^{x^2}$

$= e^{x^2} + 2x^2e^{x^2}$ [or $e^{x^2}(1+2x^2)$]

(ii) $y = \frac{\sin x}{2x}$

$y' = \frac{2x \times \cos x - \sin x \times 2}{4x^2}$

$= \frac{x\cos x - \sin x}{2x^2}$

Question 12 Continued

(b) $\int \sec^2\left(\frac{x}{2}+\pi\right) dx$

$= 2\tan\left(\frac{x}{2}+\pi\right) + C$

(c) $\int_0^2 \frac{x^3}{2+2x^4}\, dx$

$= \frac{1}{8}\int_0^2 \frac{8x^3}{2+2x^4}\, dx$

$= \frac{1}{8}\left[\ln(2+2x^4)\right]_0^2$

$= \frac{1}{8}\left[\ln(2+2(2)^4) - \ln(2+2(0)^4)\right]$

$= \frac{1}{8}\left[\ln 34 - \ln 2\right]$

$= \frac{1}{8}\ln 17$

(d) $4000, 3920, 3840, \ldots$

(i) $T_{12} = 4000 + 11 \times -80$

$= 3120$ g

(ii) $S_{12} = 4000 + 3920 + \ldots + 3120$

$= \frac{12}{2}(4000 + 3120)$

$= 42720$ g

(iii) $a = 0.75 \times 4000 = 3000$ $d = 0.75 \times -80 = -6$

$73200 = \frac{n}{2}\left[2 \times 3000 + (n-1) \times -60\right]$

$73200 = n\left[3000 - 30n + 30\right]$

$73200 = 3030n - 30n^2$

$n^2 - 101n + 2440 = 0$

$(n-40)(n-61) = 0 \Rightarrow n = 40$ $(n \leq 50)$

Question 13

(a) $4x-3y+12=0$

(i) For x intercept, $y=0 \rightarrow x=-3$ $\therefore B$ is $(-3,0)$

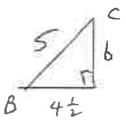
For y intercept, $x=0 \rightarrow y=4$ $\therefore A$ is $(0,4)$

$AB^2 = 3^2+4^2$

$= 25$

$AB = 5$

(ii)



$b^2 = 5^2 - \left(\frac{9}{2}\right)^2$

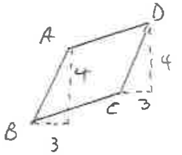
$= 25 - \frac{81}{4}$

$= \frac{100-81}{4}$

$= \frac{19}{4}$

$b = \frac{\sqrt{19}}{2}$

(iii)



$C = \left(1\frac{1}{2}, \frac{\sqrt{19}}{2}\right)$

$D = \left(1\frac{1}{2}+3, \frac{\sqrt{19}}{2}+4\right)$

$= \left(4\frac{1}{2}, \frac{\sqrt{19}+8}{2}\right)$

(b) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$3\pi - 9 = \frac{1}{2}r^2\left(\frac{\pi}{6} - \sin\frac{\pi}{6}\right)$

$3(\pi-3) = \frac{1}{2}r^2\left(\frac{\pi}{6} - \frac{1}{2}\right)$

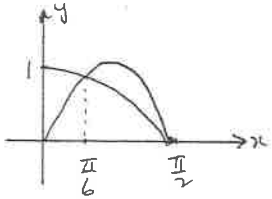
$6(\pi-3) = r^2\left(\frac{\pi-3}{6}\right)$

$36 = r^2$

$r = 6$ cm

Question 13 Continued

(c) (i)



(ii) $x = \frac{\pi}{6}$ $\quad y = \sin 2\left(\frac{\pi}{6}\right)$ $\quad y = \cos\left(\frac{\pi}{6}\right)$

$y = \frac{\sqrt{3}}{2}$ $\quad y = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{2}$ $\quad y = \sin 2\left(\frac{\pi}{2}\right)$ $\quad y = \cos\left(\frac{\pi}{2}\right)$

$y = 0$ $\quad y = 0$

(iii) $A = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x - \cos x \, dx$

$= \left[-\frac{1}{2}\cos 2x - \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \left(-\frac{1}{2}\cos\pi - \sin\frac{\pi}{2}\right) - \left(-\frac{1}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{6}\right)$

$= \left(\frac{1}{2} - 1\right) - \left(-\frac{1}{4} - \frac{1}{2}\right)$

$= \frac{1}{4}\ u^2$

Question 14

(a) (i) $P(II) = 0.35 \times 0.35$

$= 0.1225$

(ii) $P(IA \text{ or } AI) = 0.35 \times 0.55 + 0.55 \times 0.35$

$= 0.385$

Question 14 Continued

(a) (iii) $P(\text{at least one } I)$

$= P(II \text{ or } AI \text{ or } IA \text{ or } UI \text{ or } IU)$

$= 0.1225 + 0.385 + 0.1 \times 0.35 + 0.35 \times$

$= 0.5775$

(b) (i) $B = B_0 e^{-kt}$

$0.001 = 0.24 e^{-28k}$

$\frac{0.001}{0.24} = e^{-28k}$

$\ln\left(\frac{0.001}{0.24}\right) = -28k$

$k = -\frac{1}{28}\ln\left(\frac{0.001}{0.24}\right)$

$= 0.1957$ (to 4 d.p.)

(ii) $0.05 = 0.24 e^{-0.1957t}$

$\frac{0.05}{0.24} = e^{-0.1957t}$

$\ln\left(\frac{5}{24}\right) = -0.1957t$

$t = \frac{\ln\left(\frac{5}{24}\right)}{-0.1957}$

$= 8.0154\ldots$

$\therefore$ time = 11 pm + 8 hours

= 7 am

(iii) $\frac{dB}{dt} = -kB_0e^{-kt}$ $\quad (t=9)$

$= -0.1957 \times 0.24 e^{-0.1957 \times 9}$

$\doteqdot -0.008$ mg/ml

Question 14 Continued

(c) (i)

$\angle BAC = \pi - (\angle ABD + \angle BCA)$ ($\angle$ Sum $\triangle ABC$)

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$\angle ABD = \angle DAC = \frac{1}{2}\angle BAC$ (AD bisects $\angle BAC$)

$= \frac{1}{2} \times \frac{2\pi}{3}$

$= \frac{\pi}{3}$

(ii)

Area $\triangle BAD = \frac{1}{2} pq \sin \frac{\pi}{3}$

Area $\triangle DAC = \frac{1}{2} qr \sin \frac{\pi}{3}$

Area $(\triangle BAD + \triangle DAC) =$ Area $\triangle ABC$

$\frac{1}{2} pq \times \frac{\sqrt{3}}{2} + \frac{1}{2} qr \times \frac{\sqrt{3}}{2} = \frac{1}{2} pr \sin \frac{2\pi}{3}$

$\frac{\sqrt{3}\,pq}{4} + \frac{\sqrt{3}\,qr}{4} = \frac{\sqrt{3}\,pr}{4}$

$pq + qr = pr$

$\frac{pq}{pqr} + \frac{qr}{pqr} = \frac{pr}{pqr}$

$\frac{1}{r} + \frac{1}{p} = \frac{1}{q}$

Question 15

(a) $3\sin x \tan^2 x - \sin x = 0$

$\sin x\,(3\tan^2 x - 1) = 0$

$\sin x = 0$ $\qquad$ $\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$x = 0, \pi, 2\pi$ $\qquad$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(b) (i) $y = x(x-3)^2$

x intercepts at $(0,0)$ $(3,0)$

(ii) $y = x(x^2 - 6x + 9)$

$= x^3 - 6x^2 + 9x$

$y' = 3x^2 - 12x + 9$

$y'' = 6x - 12$

For T.P. $\quad 3x^2 - 12x + 9 = 0$

$3(x-1)(x-3) = 0$

$x = 1 \quad x = 3$

at $x = 1$ $\quad y = 1(1-3)^2 = 4$ $\quad y'' = 6(1) - 12 = -6 < 0$ $\cap$

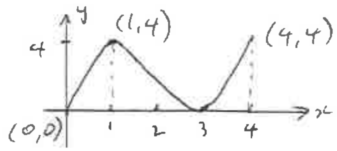
at $x = 3$ $\quad y = 3(3-3)^2 = 0$ $\quad y'' = 6(3) - 12 = 6 > 0$ $\cup$

$\therefore$ Maximum T.P. at $(1,4)$

Minimum T.P. at $(3,0)$

Question 15 Continued

(b) (iii) $y = x(x-3)^2$



(c) $S_\infty = \frac{a}{1-r}$ $\qquad a = \frac{1}{p} \quad r = -\frac{1}{p}$

$-4p = \frac{\frac{1}{p}}{1 + \frac{1}{p}}$

$-4p = \frac{1}{p+1}$

$-4p^2 - 4p = 1$

$4p^2 + 4p + 1 = 0$

$(2p+1)^2 = 0$

$p = -\frac{1}{2}$

(d) $V = \pi \int_0^{\ln 2} (e^x + 1)^2 \, dx$

$= \pi \int_0^{\ln 2} (e^{2x} + 2e^x + 1) \, dx$

$= \pi \left[\frac{1}{2}e^{2x} + 2e^x + x \right]_0^{\ln 2}$

$= \pi \left[\left(\frac{1}{2}e^{2\ln 2} + 2e^{\ln 2} + \ln 2 \right) - \left(\frac{1}{2}e^0 + 2e^0 + 0 \right) \right]$

$= \pi \left[(2 + 4 + \ln 2) - (\frac{1}{2} + 2) \right]$

$= \pi \left[\frac{7}{2} + \ln 2 \right] u^3$

Question 16

(a) (i) $a = 4\sin 2t$

$v = -2\cos 2t + C$

$t = 0,\ v = 2$: $2 = -2\cos 0 + C$

$2 = -2 + C$

$C = 4$

$\therefore\ v = 4 - 2\cos 2t$

(ii) If $v = 0$, $0 = 4 - 2\cos 2t$

$2\cos 2t = 4$

$\cos 2t = 2$

no solution $\therefore$ particle is never at rest

(iii) distance $= \int_0^4 4 - 2\cos 2t\, dt$

$= \left[4t - \sin 2t\right]_0^4$

$= (16 - \sin 8) - (0 - 0)$

$\doteqdot$ 15 metres

(b) (i) $A_1 = 400\,000(1.05) - 25\,000$

$= \$395\,000$

(ii) $A_2 = A_1(1.05) - 25000$

$= 400\,000(1.05)^2 - 25000(1.05) - 25000$

$= 400000(1.05)^2 - 25000(1 + 1.05)$

$A_n = 400\,000(1.05)^n - 25000(1 + 1.05 + \dots + 1.05^{n-1})$

$= 400\,000(1.05)^n - 25000\,\dfrac{(1.05^n - 1)}{0.05}$

$= 400\,000(1.05)^n - 500000(1.05)^n + 500000$

$= 500\,000 - 100000(1.05)^n$

Question 16 continued

(b) (iii) $0 = 500000 - 100\,000(1.05)^n$

$100\,000(1.05)^n = 500\,000$

$(1.05)^n = 5$

$n = \dfrac{\ln 5}{\ln(1.05)}$

$\doteqdot 32.986$

$\therefore$ Money runs out in 2032

(c) (i) $RB = \sqrt{x^2 + 2^2}$ $\quad BJ = 6 - x$

Time $= \dfrac{\text{Distance}}{\text{speed}}$

$T = \dfrac{RB}{8} + \dfrac{BJ}{16}$

$= \dfrac{\sqrt{x^2+4}}{8} + \dfrac{6-x}{16}$

(ii) $\dfrac{dT}{dx} = \dfrac{\frac{1}{2}(x^2+4)^{-\frac{1}{2}} \times 2x}{8} - \dfrac{1}{16}$

For minimum time $\dfrac{x}{8\sqrt{x^2+4}} - \dfrac{1}{16} = 0$

$2x - \sqrt{x^2+4} = 0$

$2x = \sqrt{x^2+4}$

$4x^2 = x^2 + 4$

$3x^2 = 4$

$x^2 = \dfrac{4}{3}$

$x = \dfrac{2}{\sqrt{3}}$

Check:

x	1	$\frac{2}{\sqrt{3}}$	2
$\frac{dT}{dx}$	-0.01	0	0.03
	\	—	/

$\therefore$ Minimum time when $BJ = 6 - \dfrac{2}{\sqrt{3}}$ km